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CHOICE OF FUNCTIONAL FORM IN THE NONLINEAR TAYLOR RULE
The Case of Pakistan

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Abstract. Linear Taylor rule prescribes symmetric response to inflation rate and output gap in good and bad times. Central banks in the world, however, are more concerned about inflation when economy is in high inflationary regime. Similarly they are more reactionary to output fluctuations when economy is experiencing slowdown in the economic activity. Thus, most of the researchers in the area of monetary policy construct a nonlinear monetary policy reaction function. In the literature related to monetary policy of Pakistan, this reaction function has been modeled as threshold regression (TR), Markov regime switching regression, and Logistic smooth transition regression (LSTR). This study compares these three choices for the case of Pakistan and tries to find out which functional form of nonlinear Taylor rule fits the Pakistani data well. Using quarterly data for the period 1993:1-2011:4, we find strong evidence that the monetary policy followed by the State Bank of Pakistan (SBP) exhibits nonlinearity. The results of this study show that threshold level of inflation rate is 6.37% and that of output gap is 2.5%. Moreover, threshold regression, with inflation rate as threshold variable, is found the best among the three specifications as it satisfies maximum number of criteria for comparison. However, LSTR model performs well if forecasting performance of the models is compared.

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I. INTRODUCTION

Monetary policy objective is to maximize society's welfare by maintaining price stability along with keeping unemployment at its natural rate. A great deal of research has been done, since the early 1990's, on monetary policy reaction functions of central banks. In particular, Taylor rule (Taylor, 1993) has received considerable attention. The rule specifies relationship between policy instrument (short term interest rate) and the target variables (inflation rate and output gap). According to this rule central banks increases the interest rate in times of high inflation, or when output is above its potential level (unemployment is below the natural rate of unemployment), and vice versa. Therefore, the rule prescribes symmetric policy action in high and low inflationary regimes.

The theoretical basis of linear Taylor rule rests on two key assumptions, namely that central banks have quadratic loss function and that the Phillips curve is linear. Recently however, both of these assumptions have been criticized. For instance, Bec *et al.* (2002), Kim, Osborn and Sensier (2002), Martin and Milas (2004), Brüggemann and Riedel (2011), Cukierman and Muscatelli (2008), Castro (2008), and Ncube and Tshuma (2010) highlight asymmetric preferences of central banks regarding inflation and the output gap, which in turn lead to nonlinear policy reaction function. Moreover, Dolado *et al.* (2004), Corrado and Holly (2003) and Nobay and Peel (2003) specify the Phillips curve relationship as nonlinear which again lead to the nonlinear policy reaction function. Dolado *et al.* (2000) relax both the assumptions and have constructed a general model which departs from linear-quadratic framework. Hence, there are good theoretical reasons to hypothesize that central banks may not be following a linear Taylor rule; empirical evidence validates this hypothesis. The nonlinear Taylor rule spells out that weights assigned to negative vs. positive output gap and low vs. high inflation rate could be different. However, we do not directly observe non-quadratic loss function or nonlinear Phillips curve so there exists unbounded universe of possible alternative nonlinear specifications of the Taylor rule.

There is limited literature and empirical work available on the monetary policy reaction function of Pakistan. In this regard, the pioneering study estimating linear Taylor rule for Pakistan is of Malik and Ahmed (2010). The study finds, using threshold regression, that State Bank of Pakistan (SBP) has never followed Taylor rule during the period 1991-2005. Ahmed and Malik (2011) find nonlinearity in the reaction function; SBP has asymmetry in the degree of leaning against the wind in high and low inflationary regimes. Saghir (2014) and Satti (2014) find instability of parameters in the monetary policy reaction function of SBP. Moreover, asymmetry is found in the response to high vs. low inflation and positive vs. negative output gap. Sattar (2014), using Markov Regime Switching framework, also depicts nonlinearity in the policy reaction function of SBP. Finally, Alam (2015) reaches the same conclusion using Logistic Smooth Transition model.

The nonlinearity in the policy reaction function, once established, becomes part of the macroeconomic models analyzing monetary policy issues. But question remains how nonlinearity should be modeled; threshold regression, Markov regime switching framework, or smooth transition regression model. We believe that it is important to investigate the type and nature of nonlinearity in SBP's reaction function while avoiding specific parametric assumptions. Once we are out of the realm of linear framework, the specification problem has to be addressed. Adopting an incorrect nonlinear specification is more problematic than simply ignoring the nonlinearity altogether.

Our main contribution is to determine the most appropriate form of nonlinearity in the policy reaction function of SBP. In this regard, we compare the results of three models; Threshold regression model, smooth transition regression model and Markov regime-switching model. These three models differ on the basis of their mode of transition from one state to another. In threshold regression model, parameters abruptly change from one regime to another regime implying sharp threshold while smooth transition regression allows for the smooth and gradual transition of the parameters from one state to the other. In Markov regime switching model there is exogenous regime switching having fixed probabilities.

In this study, we have first estimated the simple linear Taylor rule which did not fit the data well. Therefore, we have estimated nonlinear Taylor rule with the three potential nonlinear techniques, *i.e.* Threshold regression model, Smooth Transition regression model and Markov regime switching model. The objective of our study is to compare these models and to find the best fitted model for our data. The three models are compared on the basis of six criteria, *i.e.* Akaike information criterion, Shwartz information criterion, coefficient of determination, coefficient of correlation, root mean square error and mean absolute error.

The structure of rest of the study is as follows. The following section presents a review of the relevant literature. Section III describes in detail the nonlinear econometric models and their comparison techniques. Moreover, details of data and variables are also given in this section. Estimation results are then presented and discussed in section IV, and section V concludes the study.

II. LITERATURE REVIEW

Monetary policy is the demand side macroeconomic plan of action or strategy set by the nation's central bank in order to achieve the macroeconomic goals which are achieved by manipulating money and credit supplies or by changing interest rates. The idea of monetary policy originated in 1699 when the Bank of England printed notes backed by gold. Later on during 1870-1920, the developed or industrialized nations set up central body known as central bank for laying the monetary policy. The objectives of monetary policy may vary across countries but the main objectives do not change which are controlling inflation rate, exchange rate stability and stabilization of economic activity. For the monetary stability, there are policy tools like open market operations, discount window borrowing and reserve requirements. The operating target of monetary policy is set either by a pre-specified rule or it remains discretionary choice of the central bankers.

Simons (1936) was the first to raise the issue regarding the rules vs. discretion of monetary policy and favoured policy rule for the economic stability. Discretion is authorization to enhance economic performance whereby actions are done solely on the basis of judgment whereas rule is considered a constraint. Monetary policy rule has been advocated against discretion by the prominent economists including Kydland and Prescott (1977), Fischer (1980), Barro and Gordon (1983), McCallum (1988) and Taylor (1993).

The idea of rule as a practical guide for monetary policy was popularized by McCallum (1988) and Taylor (1993). McCallum proposed changes in money growth rate in response to changes in inflation rate and GDP growth rate. Taylor rule prescribes changes in short term interest rate in response to changes in inflation rate and output gap; the relationship is assumed to be linear. The assumptions of quadratic loss function and linear Phillips curve lead to linear and symmetric response of central bank to inflation deviation from the target and output deviations from potential level.

Linear Taylor rule has been criticized by many intellectuals on the basis of its assumptions and once any of these assumptions is relaxed monetary policy response function becomes nonlinear. Central bankers' preferences regarding stabilization of economic activity and inflation rate are modeled symmetric, perhaps due to mathematical convenience, but actually these preferences might be asymmetric either due to the bankers' own choice or due to political pressure (Blinder, 1999). Policy makers tend to take more serious actions when output is below its potential (unemployment is higher) and/or inflation rate is above its target. The response to deviations is less severe when output is above its potential (unemployment is lower) and/or inflation rate is below its target. This kind of behaviour is quite close to the human psychology as human beings try to avert loss while welcome bliss, expansion and benefits. Moreover, the shape of Phillips curve is found convex instead of linear,[1] which shows that at any point on the curve the increase in inflation rate is more in order to decrease the unemployment than decrease in inflation rate when unemployment of same magnitude gets increased. Inflation responds strongly to excess demand in expansion while it gets insensitive to the output during recession (Laxton *et al.*, 1999). The convexity of Phillips curve is also supported by the downward wage rigidity. Optimal monetary policy response also gets nonlinear when Phillips curve is nonlinear. For instance, Dolado *et al.* (2005) empirically tested the convexity of Phillips curve for four European countries where labour market rigidities were severe and derived the nonlinear policy rule. Tambakis (1998) and Corrado and Holly (2003) find that, in the presence of nonlinear Phillips curve, positive inflation bias (average inflation exceeds the target) has been observed if linear rule is specified. On the other hand, Nobay and Peel (2003) find deflation bias in output gap and do not find any significant signs of inflation bias while deriving optimal monetary policy reaction function. Due to these two reasons monetary policy reaction function is modeled and estimated as nonlinear (*see* for instance, Bec *et al.*, 2002; Kim, Osborn and Sensier, 2002; Martin and Milas, 2004; Brüggemann and Riedel, 2011; Cukierman and Muscatelli, 2008; Castro, 2008; Ncube and Tshuma, 2010). Moreover, type of asymmetric preferences lead to different nonlinear shapes of the reaction function; when there is recession avoidance preference, the reaction function happens to be the concave with respect to the output gap while inflation avoidance preference leads to the convex reaction function with respect to the inflation gap (Cukierman and Muscatelli, 2008).

Linear estimation techniques cannot be applied to analyze the features of nonlinear behaviour. Thus, nonlinear behaviour should be handled with nonlinear specification and the model should be estimated using nonlinear

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[1]Stiglitz (1997) however, talked about concavity of the Phillips curve.

estimation techniques. The idea behind the nonlinear models is “regime shifts or regime switching” and these models were first introduced by Goldfeld and Quandt (1972). There are three types of nonlinear specifications of the regression models. First is the threshold regression model, developed by Tong (1983), in which change of behaviour of a variable has been observed above and below the certain value or set of values of a threshold. Regime shift in this model is assumed to be discrete and can be determined endogenously from the data. This model can be easily estimated by Ordinary Least Square Technique (*see* for instance, Komlan, 2013; Koustas and Lamarche, 2010; Rodriguez, 2008). Second, the smooth transition regression models which let the parameters to change smoothly and slowly from one regime to another and allows for endogenous regime switches. Smooth transition autoregressive (STAR) models have two useful types; logistic version of star model (LSTAR) and exponential form of the model (ESTAR). Castro (2008) and Peterson (2007) argue in the favour of using this type of models as these provide better structural framework and economic intuition for the central banks’ nonlinear policy behaviour.[2] Third, Markov switching model developed by Hamilton (1989) captures the nonlinear behaviour of monetary policy assuming regime switching a Markov process. Several authors have followed this approach. Tan and Habibullah (2007) empirically assessed the asymmetric behaviour of monetary policy with the business cycles using this technique. Yi (2012) argues in favour of using this technique to the ‘crisis mentality’ of Asian emerging economies. Owyang and Ramey (2001) point to the presence of ‘dove’ and ‘hawk’ regimes in the monetary policy of US, using this methodology.

Before deciding for the appropriate form of the nonlinearity, detection of the nonlinear behaviour is a pre-requisite. If nonlinearity is not detected, applying a nonlinear model can lead to over fitting the data. This can be done with number of tests like the McLeod test, RESET test, LM test, and other portmanteau test. McLeod-Li (1983) test is done to determine if there is presence of significant autocorrelations in the squared residuals from linear equation. Ljung-Box statistic is used to determine serial correlation in the squared residuals. If the null hypothesis is rejected, this shows that the model is nonlinear. This test can detect various forms of nonlinearity but cannot specify the actual nature and form of the nonlinearity present in the data. Regression Error Specification Test is used to test the linearity of the model.

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[2]Using the LSTAR specification, Huh, Lee and Lee (2009) models the nonlinear Phillips curve of US to inquire optimal policy rule and Brüggemann and Riedel (2011) models quarterly data of UK which according to them was most plausible and viable technique.

Thus, null hypothesis is linearity and the alternative hypothesis is nonlinear specification of the model. The concept of doing this test relies on the fact that for the linearity to hold, the residuals of the estimated linear model should be uncorrelated to the regressors used in the estimating equation or with the fitted values. The advantage of using this test is that it's easy to apply and detect nonlinearity. Other portmanteau tests are residual-based while having no specific alternative hypothesis. One of these tests is BDS (Broock *et al.*, 1996) test for independence. Distance between different pairs of residuals has been examined in this test for the detection of serial correlation, nonlinearity, and structural breaks. McLeod-Li Test, the RESET and other Portmanteau tests provide little help in determining the nature of the nonlinearity as these tests have general alternative hypothesis of nonlinearity. To tackle this issue, Lagrange Multiplier Test is used since it has specific alternative hypothesis. This test can be done in three steps where residuals of the estimated linear portion model are regressed on the partial derivatives estimated under the null hypothesis of linearity and then on the basis of the value of TR^2, which has χ^2 distribution, is used to accept or reject the null hypothesis of linearity.

The focus of our study is nonlinear Taylor rule with reference to State Bank of Pakistan. By reviewing the literature related to Pakistan's monetary policy (given in introduction), it is found that monetary policy reaction function in Pakistan is asymmetric. The asymmetric response in Pakistan is modeled, in different studies, as threshold regression, Logistic smooth transition regression and Markov regime switching models. However, we have analyzed with the help of literature that these techniques vary on the basis of the type of the 'regime switching'. Our objective in this study is to find the most appropriate specification of the nonlinear Taylor rule in Pakistan.

III. ECONOMETRIC METHODOLOGY

The purpose of our study is to find out the appropriate specification of monetary policy reaction function of SBP to capture nonlinearity. Initially linear Taylor rule is estimated and then different nonlinear specifications are tested.

LINEAR MODEL

The linear static Taylor rule can be specified as the following equation:

$$i_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + u_t \tag{1}$$

Here, i is the short term nominal interest rate, π is the inflation rate and y is output gap. Coefficients α_0 depends on the inflation target and equilibrium real interest rate, α_1 is assumed to be positive and greater than 1 (1.5 in Taylor's specification) and α_2 is assumed to be positive for policy to be counter-cyclical. u_t is the error term which is identically distributed but may be serially correlated.

The serial correlation of the error term is indicative of the interest rate smoothing objective which is one of the objectives of SBP (Malik, 2007). In this case the above specification of Taylor rule is inappropriate. If we correct the above specification for serial correlation then it becomes dynamic version of the linear Taylor rule.

$$i_t = \rho i_{t-1} + (1-\rho)(\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t) + e_t \tag{2}$$

In this case ρ is the first order autocorrelation coefficient and e is assumed to be serially uncorrelated error term.

NONLINEAR MODELS

Nonlinear econometric models are based on the concept of regime switching where behaviour of the variable depends on the state of the economy. There are different kinds of regime switching models which are discussed in this sub-section.

1. Threshold Regression Model

In this type of model response of a variable changes above and below the certain threshold value or set of values. Threshold regression model, developed by Tong (1983) and Chan and Tong (1986), is quite useful in the field of economics for the analysis of the nonlinear models. This model can be easily estimated by Ordinary Least Square Technique (OLS). In order to capture the nonlinearity and asymmetry in the Taylor rule, we used the following TR model:

$$i_t = \rho_1 i_{t-1} + (1-\rho_1)(\alpha_{01} + \alpha_{11}\pi_t + \alpha_{21} y_t) + e_{1t} \quad \text{if } \pi > \tau \quad \text{and}$$

$$i_t = \rho_2 i_{t-1} + (1-\rho_2)(\alpha_{02} + \alpha_{21}\pi_t + \alpha_{22} y_t) + e_{2t} \quad \text{if } \pi \leq \tau \tag{3}$$

The above equation represents threshold model with two regimes defined by the value of inflation where threshold inflation rate is τ, above and below which parameters' values are different. The process is linear in each regime but the possibility of switching from regime 1 to regime 2 renders Taylor rule a nonlinear process. Estimation of threshold model is performed by OLS technique and it is easier to estimate if threshold τ is known. The threshold value, if unknown, can be estimated using Chan (1993) methodology. In our case, we have 72 observations, the maximum and minimum 13% of the values are trimmed to ensure enough values on each side of the threshold value. Exclusion of 26% from both the sides leaves us with 52 values. The above equation (3) has been estimated 52 times considering each of the remaining observations as potential threshold value. The regression with the smallest sum of square of residuals contains the estimate of threshold. With similar methodology threshold for output gap is also determined in this study.

2. Smooth Transition Regression Model

Parameters change slowly from one regime to another in STR model capturing smoothness of the regime switching in monetary policy reaction function. Following Teräsvirta (1994; 1996), STR model for nonlinear monetary policy reaction function can be defined as follows:

$$i_t = \alpha' z_t + \gamma' z_t G(\gamma, c, s_t) + \varepsilon_t \tag{4}$$

Where z_t represents the vector of the explanatory variable which includes i_{t-1}, π_t, and y_t. Vectors ε' and γ' include the parameters associated to the linear and nonlinear parts of the equation respectively. The error term ε_t is assumed to be normally distributed with zero mean and constant variance. $G(\gamma, c, s_t)$ is known as transition function which is continuous and bounded between zero and one depending on the smoothness parameter γ, location parameter c and transition variable s_t, which may be an independent variable or a linear combination of the elements of z_t.

There are two basic transition functions in this case; the Logistic Smooth Transition Regression model and the Exponential Smooth Transition Regression model.

(LSTR) model has the following transition function:

$$G(\gamma, c, s_t) = [1 + \exp\{-\gamma(s_t - c)\}]^{-1} \quad \gamma > 0 \tag{5}$$

This function is monotonically increasing function of transition variable s_t. γ is the smoothness parameter indicating how smoothly the transition occurs between the regimes while c is the location parameter indicating where transition has actually taken place.

ESTR model has the following exponential transition function:

$$G(\gamma, c, s_t) = 1 - \exp\left[-\gamma(s_t - c)^2\right], \quad \gamma > 0 \tag{6}$$

3. Markov Regime Switching Technique

Markov regime switching model developed by Hamilton (1989) posits that regime switches are exogenous Markov processes. Our model is specified in a fashion that there exists two possible regimes for each target variable .The Taylor rule specification for Markov switching process is as follows:

$$i_t = \alpha_0\left(S_t^m\right) + \alpha_n\left(S_t^m\right)\pi_t + \alpha_{i_{t-1}}\left(S_t^m\right)i_{t-1} + \alpha_y\left(S_t^m\right)y_t + \sigma_i\left(S_t^m\right)\varepsilon_t^i \tag{7}$$

S_t^m is the monetary policy regime and $\varepsilon_t \sim N(0,\sigma^2)$. There exist fixed probabilities of regime changes. If p_{11} denotes the probability that the system remains in regime one then $(1 - p_{11})$ denotes the probability that the system switches from regime 1 to regime 2. Similarly p_{22} denotes the probability that the system remains in regime two and $(1 - p_{22})$ denotes the probability that system switches from regime 2 to regime 1. S_t is unobservable in the data so we can only make inferences about the state based on the Markov transition probabilities. The coefficients of the two regimes and their transition probabilities are estimated through Maximum Likelihood Estimation (MLE) method. The transition probabilities are conditional probabilities and they are unknown, so they have to be estimated along with the coefficients of the model.

MODEL SELECTION CRITERIA

Our aim is to find the most appropriate model by comparing results from different models on the basis of following criteria:

1. The Information Criteria

We used information criteria, such as Akaike Information Criterion (AIC) and Shwartz Information Criterion (SIC) for selecting the appropriate model. An advantage of these information criteria is that they can be used to compare non-nested models. AIC provides a relative estimate of the information loss when a certain model performs a data generating process. There is always possibility of some information loss as one of the candidate models is used to represent the “true” model.

Generally, AIC is estimated as:

$$AIC = 2k - 2\ln(L) \tag{8}$$

Here, *k* represents the number of parameters and *L* is the maximized value of the likelihood function of the estimated model. That model is preferred from a set of models which has the minimum AIC value.

SIC is also based on the likelihood function. The general formula for SBC is as follows:

$$SIC = -2.\ln \hat{L} + k.\ln(n) \tag{9}$$

Where $\hat{L}$ represents the estimated log likelihood value, k is the number of parameters and n is the sample size. The model with lower SBC is the one to be preferred. This criterion has lower probability of over-fitting the data than AIC.

2. Coefficient of Determination (R-square)

The most general definition of coefficient of determination is

$$R^2 = 1 - \frac{SSR}{SST} \tag{10}$$

Where SSR and SST are the residual sum of squares and total sum of squares respectively. It is useful for model selection when the specifications differ on the basis of addition or deletion of the explanatory variables (Johnston and Dinardo, 1997).

3. Forecasting

Forecasting helps to predict about the economic conditions, so it is important to find how well the four models under study perform in forecasting. For forecasting accuracy, the following statistical tools are used in this study.

Root Mean Square Error

It is the measure of the difference between the values predicted by the estimated model and the values actually observed. It compares the forecasting errors of different models. It can only have positive values; the model with a smaller value of RMSE is the better one to diagnose the variation in the errors in a set of forecast. Formula to calculate RMSE is as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{n}} \tag{11}$$

Where y_t and $\hat{y}_t$ represent actual and predicted values and n is the number of observations.

Mean Absolute Error

This measure of forecast error indicates how close forecasts are to eventual outcomes. MAE is given by:

$$\frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{12}$$

Where e_i is the residual obtained from the estimation and n is the number of observations.

Coefficient of Correlation

The coefficient of correlation between the actual and the fitted values indicates the degree of linear association between these two series. The high value indicates the better fit of the model from which forecasts are generated.

DATA AND VARIABLES

For estimation of the monetary policy reaction function of SBP we have used quarterly data over the period 1993Q1-2011Q4. Before this time period State Bank of Pakistan was not given the autonomy to set operating target. Therefore the time span chosen starts from 1993. SBP achieves its objectives by targeting KIBOR (Karachi Interbank Offered Rate) at midpoint of the *repo* rate corridor. Though State Bank of Pakistan uses KIBOR as a policy instrument but due to unavailability of its data over the period under consideration we have used Call Money Rate as its proxy. Output gap is the difference between the actual output of the economy and the potential output. For the construction of this variable, data on GDP (constructed using the methodology of Arby (2008)) was seasonally adjusted by four quarters moving average method. The annual data on GDP have been collected from Pakistan Economic Survey. In Pakistan data on GDP are revised twice after release of provisional data, therefore, only provisional data are available for the last two years. The seasonally adjusted quarterly GDP is then regressed on time and time square; the resulting fitted values are used as proxy of the potential GDP. The output gap is estimated as the percentage difference of seasonally adjusted GDP and the potential level of GDP. Inflation rate is calculated as year on year growth rate of quarterly values of consumer price index (CPI), data on which are obtained from International Financial Statistics (IFS).

IV. ESTIMATION RESULTS

In the first step linear model has been estimated to find whether or not SBP follows linear Taylor rule. After getting the evidence of nonlinearity, all the three regime switching nonlinear techniques have been applied. Later on comparison is made among the models to find which one is the most appropriate description of nonlinear behaviour of monetary policy reaction function of Pakistan.

LINEAR TAYLOR RULE

Table 1 reports the estimation results of our static and dynamic versions of linear Taylor rule. The results clearly show that SBP has not been following linear Taylor rule as our coefficients' estimates are different to what have been taken by Taylor. It is worth noticing that all the coefficients are statistically significant and residual series of the estimated rule is found stationary. However, the values of Durbin Watson (DW) stat and adjusted R-square are low which indicate other objectives of monetary authority in Pakistan. To capture the effect of another objective – interest rate smoothing – lagged interest rate is introduced on right hand side of the Taylor rule. Dynamic version of Taylor rule also tackles the problem of autocorrelation.

TABLE 1

Estimation Results of Linear Taylor Rule

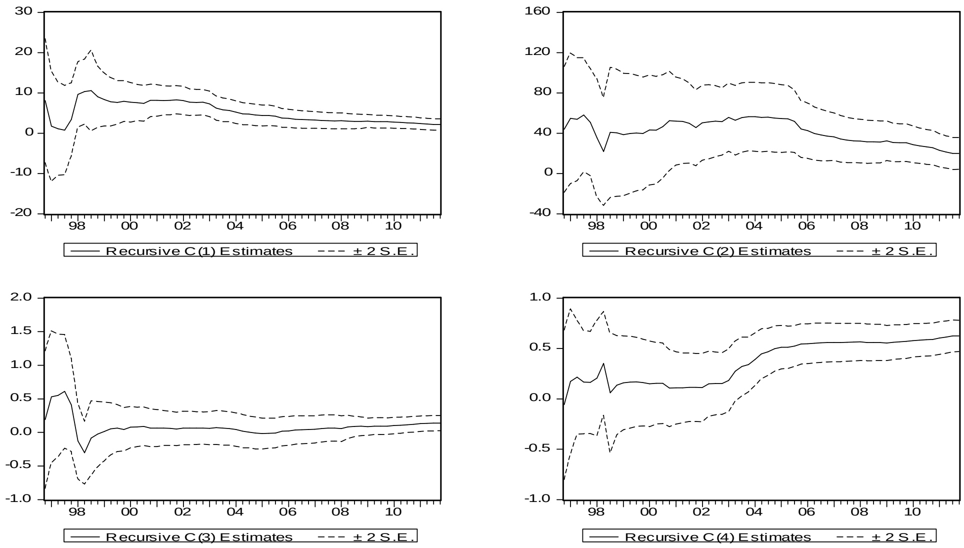
Variables	Static Taylor Rule		Dynamic Taylor Rule	
	Coefficient Estimates	P-values	Coefficient Estimates	P-values
Constant	6.02	0.0000	2.15	0.003
Output Gap	28.77	0.0092	19.84	0.01
Inflation Rate	0.33	0.0000	0.137	0.01
Lagged Interest Rate			0.62	0.000
Adjusted R-Square	0.4		0.688	
DW-stats	0.58		2.06	
F-stats	24.18	0.000	53.28	0.000

NOTE: F-stats are estimated for the null hypothesis that all coefficients except constant are zero.

It is worth noticing that coefficient of the lagged interest rate is statistically significant and high in magnitude indicating interest rate smoothing objective of the SBP. Output stabilization is another objective of monetary policy in Pakistan but coefficient of inflation rate is less than 1 in both specifications. We have also found recursive estimates of the coefficients in dynamic version of the Taylor rule to have an idea of the stability of the parameters. The estimates of coefficients of output gap [C(2)] are unstable changing in 2007 when economy went into the problem of stagflation. At that time coefficient of inflation rate [C(3)] was also increasing but this increase was less than the increase in inflation rate. The coefficient of lagged interest rate [C(4)] significantly increased after 2007 as SBP continuously increased discount rate after this time period.

FIGURE 1

Recursive Estimates of Coefficients in Dynamic Taylor Rule



NONLINEAR TAYLOR RULE

1. Threshold Regression Model

In the first step, threshold values of inflation and output gap are estimated. For this Chan's (1993) method has been used. The values of sum of the squared residual obtained from the 52 regressions of the threshold model are plotted against successive values of threshold variable. The graph shown in Figure 2(a) clearly demonstrates that there is a sharp trough at fourteenth observation, which indicates viability of the threshold regression model. Threshold inflation rate has been found to be 6.37%. Same procedure is repeated for output gap; the threshold value of output gap is found 0.025 (2.5%). Hence, there is possibility that SBP's response to deviations of inflation from target change when inflation rate crosses the threshold value of 6.37%. Similarly, the response may be different above and below 0.025 value of output gap.

FIGURE 2(a)

Residual Sum of Squares to Find Threshold Inflation Rate

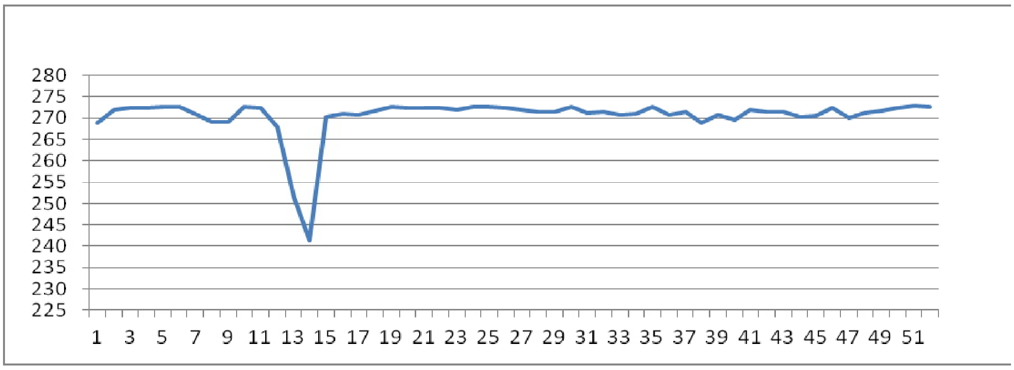
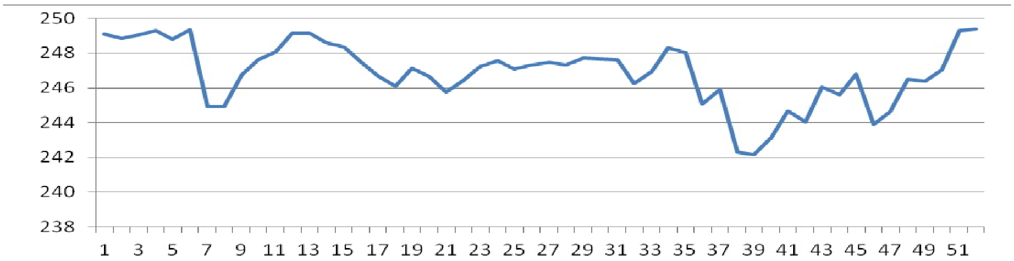


FIGURE 2(b)

Residual Sum of Squares to Find Threshold Output Gap



The next step is to estimate threshold regression for monetary policy analysis. Results from Table 2 show that all the coefficients are statistically significant (except for constant) when threshold inflation rate is used to construct dummy variable, which is then multiplied with the inflation rate. The fit of the model improved a lot as adjusted R-square is found 0.73 and Durbin-Watson stats is also close to 2. The magnitude of coefficient of output gap is still high and that of lagged interest rate is almost same as found in linear Taylor rule. The coefficient of inflation rate is different to what has been found in case of linear Taylor rule. Moreover, the coefficient of inflation rate is different in high and low inflationary regimes. This coefficient is greater than 1 (after adjusting for the coefficient of lagged interest rate) only when inflation rate is less than threshold inflation rate. Wald test shows that coefficients of inflation rate in two regimes are statistically different from each other. This shows that SBP is able to effectively respond to inflation rate only when economy is in low inflationary regime. Similarly in case, when regimes are defined with respect to output gap, coefficient of output gap is higher and statistically significant when output gap is below its threshold value. It means response to output gap movements are also effective only when the economy is under performing. This behaviour is also confirmed when interaction dummy variables are used; SBP effectively responds to output gap only when output gap is negative and inflation rate is below threshold (Results are given in Appendix).

TABLE 2

Estimation Results of Threshold Regression

Variables	Regimes with respect to Inflation Rate		Regimes with respect to Output Gap	
	Coefficient Estimates	P-values	Coefficient Estimates	P-values
Constant	0.60	0.44	2.6	0.001
Output Gap	24.8	0.001		
Inflation Rate			0.14	0.015
Lagged Interest Rate	0.61	0.000	0.6	0.000
Inf * Dum_Inf	0.24	0.000		
Inf * (1-Dum_Inf)	0.71	0.000		
Gap * Dum_gap			6.5	0.58
Gap * (1-Dum_gap)			33.6	0.008
Threshold Inflation Rate	6.37		0.025	
Threshold Output Gap				
Adjusted R-square	0.73		0.69	
DW stat	1.87		2.02	
F-stat	49.00	0.000	41.02	0.000
Wald Stat	11.49	0.000		

F-stat is estimated for the null hypothesis that all coefficients except constant are zero.

2. Results of Markov Regime Switching Model

In order to estimate nonlinear Taylor rule, five specifications are used in Markov regime switching model. In the first specification, all policy parameters including intercept and error variance are regime variant. In second specification only coefficient of inflation is regime variant keeping all other parameters constant across the regimes. Contrary to this, in third specification, only coefficient of inflation is regime invariant keeping all other parameters as regime variant, while in the fourth specification coefficient of gap is switching keeping all other parameters as regime invariant. In the last specification, coefficient of lagged interest rate is variant while keeping all other parameters as non-switching and invariant to regime change. The results are shown in Table 3.

TABLE 3

Estimation Result of Markov Regime Switching Model

Parameters	Specification 1		Specification 2		Specification 3		Specification 4		Specification 5	
	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
α_0	1.05 (1.00)	7.05 (1.00)	0.00 (1.00)		7.89 (1.00)	1.05 (0.00)	0.00 (1.00)		0.00 (0.00)	
α_{it-1}	0.77 (0.00)	2.39 (1.00)	0.60 (1.00)		0.1 (1.00)	0.76 (0.00)	0.78 (1.00)		0.82 (0.00)	–0.4 (0.00)
α_π	0.12 (1.00)	0.03 (1.00)	0.87 (0.00)	–0.87 (1.00)	0.11 (0.00)		0.1860 (1.00)		0.15 (0.00)	
α_y	9.93 (1.00)	47.52 (1.00)	24.24 (1.00)		36.23 (0.00)	12.8 (1.00)	11.75 (0.00)	139.56 (1.00)	8.11 (0.00)	
σ_i^2	0.6 (1.00)	6.24 (1.00)	11.53 (1.00)		5.82 (0.00)	0.89 (1.00)	3.73 (0.00)		4.47 (0.00)	
Expected regime duration	8.18	4.17	10	10	12.11	44.47		5.82		1.00
Log Likelihood	–140.81		–282.19		–138.82		–157.73		–162.62	

The first specification is standard for the monetary policy rule as used by Davig and Leeper (2006); results demonstrate that SBP does not follow Taylor rule as coefficient of inflation is 0.12 in low variance state and 0.03 in high variance state. These results are consistent with Malik and Ahmed (2010) who find that Taylor rule has never been followed by SBP. In the second specification, only coefficient of inflation rate is significant (when economy is in low variance state). In the third specification, coefficient of inflation rate is significant but its magnitude is low while coefficient of output gap is significant only in high variance state. All specifications show that State bank of Pakistan has never followed Taylor rule in both volatile and docile periods as coefficient of inflation rate is less than one in all the cases. Monetary policy is active (in Leeper's terminology) when it satisfies the Taylor principle, that is, estimated coefficient of inflation is greater than one, and in periods when it is passive, the same coefficient is less than one. Our results show that SBP has passive policy stance.

TABLE 4

Results of LSTR Model (with i_{t-2} as Transition Variable)

Parameters	Linear Part		Nonlinear Part	
	Coefficient Estimates	P-values	Coefficient Estimates	P-values
Constant	2.32	0.03	3.97	0.02
Output Gap	30.7	0.01	–29.3	0.06
Inflation Rate	0.06	0.54	0.1	0.42
Lagged Interest Rate	0.54	0.00	–0.27	0.11
Alpha			8.4	0.00
Gamma			4.74	0.35
DW stats	1.85			
R-square	0.77			

3. Results of Smooth Transition Regression

First of all, we have tested linearity against STR model using residual based LM test which is discussed in Teräsvirta (1996). Linear Taylor rule is estimated and then the residuals from this linear model are used to estimate following auxiliary equation:

$$\hat{\varepsilon}_t = \delta_0' w_t + \delta_1' w_t s_t + \delta_2' w_t (s_t)^2 + \delta_3' w_t (s_t)^3$$

w_t denotes the vector of explanatory variable while s_t denotes the transition variable. The null hypothesis of linearity is H_0: $\delta_1 = \delta_2 = \delta_3 = 0$. Linearity is tested for several transition variables including lags of the explanatory variables and second lag of interest rate is chosen on the basis of its lowest p-value of χ^2 which is 0.005 for the rejection of the linear model. Moreover, further testing of the cubic expressions of the above equation shows that LSTR model, in comparison with the ESTR model, is more appropriate for the monetary policy reaction function of SBP. Results of LSTR model indicate that coefficient of inflation rate is insignificant in linear as well as in nonlinear part of the model. Moreover, it is found that SBP is concerned with output stabilization rather than price stabilization. Output gap has the positive sign in the linear part but opposite sign in the nonlinear part implying that the coefficient's value is decreasing with the increase in interest rate (in high inflationary regime). The coefficient of lagged interest rate is significant only in the linear part but not in nonlinear part indicating that the inertia coefficient decreases with increase in the lagged interest rate. Location parameter, alpha is significant while smoothness parameter gamma is insignificant in our results.

TABLE 5

Comparing Different Specifications of Nonlinear Taylor Rule

Model	AIC	SBC	R^2	RMSE	MAE	r
Linear rule	4.14	4.31	0.70	2.30	1.92	0.83
TR(inf)	4.06	4.21	0.74	2.22	1.80	0.86
TR(gap)	4.18	4.34	0.71	2.26	1.87	0.84
LSTR	10.1	32.9	0.22	1.60	1.15	0.47
MS(spec1)	18.1	49.9	0.40	1.75	1.27	0.63
MS(spec2)	8.71	31.4	0.05	4.31	3.49	0.22
MS(spec3)	16.1	45.7	0.38	1.71	1.22	0.61
MS(spec4)	9.87	32.6	0.07	1.97	1.38	0.28
MS(spec5)	9.81	32.5	0.08	1.99	1.35	0.29

RESULTS OF MODEL SELECTION CRITERIA

The selection of the most suitable model is based on the different criteria which are shown in Table 5. Results demonstrate that threshold regression model with inflation as a threshold variable has the minimum value of AIC and SBC while the Markov regime switching specification having all the parameters as regime variant has highest values of these criteria. Coefficient of determination, R^2 has the highest value in threshold regression model with inflation rate as threshold variable and Markov switching model with only inflation's coefficient as regime variant has its lowest value. LSTR model has the lowest standard deviation of the unexplained variance and closest fitted values to the actual outcomes as shown by the lowest values of RMSE and MAE. The coefficient of correlation between the actual and the predicted values is highest for the TR model with inflation as transition variable. These results imply that on the basis of information criteria, R^2 and correlation coefficient between predicted and actual values, TR model with inflation as the threshold variable is the best model but if one is only interested in forecasting performance of the model then LSTR gives the minimum RMSE and MAE. The choice between these two models, *i.e.* TR model with inflation as threshold variable and LSTR model depends on the researcher's objectives.

V. CONCLUSION

According to Linear Taylor rule, monetary policy reaction function of Pakistan would set its interest rate as a policy instrument in response to inflation and output gap. This shows that whatever the inflation rate or the output gap is in the economy, policy makers would behave symmetrically to bring these economic indicators to their respective target levels. Doesn't it sound unrealistic? Human psychology and real life aggregate supply function lead to asymmetric response of policy makers. Would a policy maker treat positive and negative output gap and high and low inflation the same way? No, it rarely happens in the real world where loss aversion is the prevailing characteristic. Our estimation results of static linear Taylor rule and dynamic version of linear Taylor rule clearly indicate that SBP has never followed a linear Taylor rule. This convinces us to widen our research out of the realm of linearity.

After getting to know that response function is nonlinear, its proper specification is the major area of concern as misspecification of nonlinearity is a bigger problem than ignoring it altogether. "What is the type and nature of nonlinearity in the Taylor rule of Pakistan?" To find the answer, we estimated the reaction function using three different nonlinear models namely the Threshold regression model, logistic smooth transition regression model and Markov regime switching model. These three models have 'regime switching behaviour' but they differ on the basis of the mode of switching of parameters between the regimes. This characteristic makes each model unique in its specification and tells about the nature of nonlinearity. Later on, to find the answer of our research question, we have to choose between the models to know the nature of the nonlinearity in the reaction function of Pakistan. This can be done by comparing the estimated models on the basis of best-fitted criteria. The model satisfying the maximum criteria is the one to be used to estimate reaction function of Pakistan.

First of all we estimated Threshold regression model and found that threshold level of inflation rate and output gap were 6.37% and 2.5% respectively. These thresholds act as pivots leading to low and high inflationary regimes and good and bad times. Our results show that SBP responds to output gap in all specifications but reaction to inflation rate is significant and coefficient of inflation rate is greater than 1 only in low inflationary regime. We also estimated logistic smooth transition regression model and found that SBP does not target inflation rate in Pakistan rather it is concerned with output stabilization and interest rate smoothing. In the Markov regime switching model we find that SBP has never followed Taylor rule and Taylor principle is not satisfied (hence, monetary policy is passive). All these models have common finding that SBP desires to smooth interest rate changes over time. We conclude from the comparison of results from different models that best fitted model is threshold regression model with inflation as threshold variable according to which SBP reacts asymmetrically above and below threshold inflation rate of 6.37%. We can conclude from this result that high inflation usually accompanies high level of output therefore SBP does not raise interest rate to high levels to curb the output and inflation deviations from their respective targets. It can also be related to the fact that SBP faces political pressure when it raises interest rate during expansions.

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APPENDIX

TABLE A1

Categorization of Dummy Variables of Output and Inflation Rate

DBHINF	DBLINF	DRHINF	DRLINF
Good Time and high inflation = 1, otherwise 0.	Good Time and low inflation = 1, otherwise 0.	Bad Time and high inflation = 1, otherwise 0.	Bad Time and low inflation = 1, otherwise 0.

TABLE A2

Regression Results (With Dummy Variables Representing Four States of the Economy)

Constant	2.91 (0.0079)*
Gap*DBHINF	12.8 (0.53)
Gap*DBLINF	51.22 (0.29)
Gap*DRHINF	–33.7 (0.39)
Gap*DRLINF	52.78 (0.009)*
Infl*DBHINF	0.102 (0.22)
Infl*DBLINF	–0.104 (0.7)
Infl*DRHINF	0.033 (0.71)
Infl*DRLINF	0.192 (0.36)
Cmr(–1)	0.6 (0.000)*
Threshold Output gap	2.5%
Threshold inflation rate	6.37%
Adjusted R-square DW stat F-stat	0.68 1.96 18.35 (0.000)*

NOTE: The values in the parenthesis are p-values. The * indicates significance at 1%. Dependent variable = Call Money Rate